Parametric modal testing using slow but continuous variation of operating conditions. Illustration on a contact bench.

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Abstract

Many systems have vibration properties, modes and non-linear limit cycles, that are significantly affected by environmental properties such as temperature, pressure, rotation speed, ... The classical testing strategy is to select a number of fixed operating conditions, wait for the system to stabilize at each condition and extract the vibration characteristics. A lot of test configurations may thus be required to finely characterize the influence of environmental parameters on the system dynamics. The novel point of view taken here is to consider that in a number of applications, one can generate a slow variation of the operating condition that is sufficiently below the frequencies of interest to allow continuous monitoring of vibration properties.

The application chosen as illustration of the process is a test bench seeking to characterize contact stiffness properties in a brake subassembly focusing on piston/backplate and piston/chamber contacts. A first rough estimate of the relation between modal frequency and pressure is obtained by traditional modal tests. A series of tests then changes pressure and excitation frequency continuously seeking to track phase resonance and thus update the estimates of resonance frequency, damping and shape changes at a very large number of pressure points. The proposed methodology using a feed-forward approach results in tests that are inherently certain to provide a result in a controlled amount of time.

1 Introduction

Experimental modal analysis [1, 2] is the main approach used to characterize the vibrations of dynamic systems, with a wide range of industrial and academic applications. It assumes linear time invariant system allowing the identification of modes from the measured transfer functions obtained from the measured response to an measured excitation.

Most real systems however have a behavior that is dependent on environmental parameters and operating conditions that need to be considered. For example, it is well known that material properties change with temperature and that changes on static loads affect the behavior of contact surfaces. The usual approach in such cases is to discretize parametric space in a set of operating points and characterize an equivalent linear behavior at these points, and use interpolation to create a linear parameter-varying system (LPV). One notable example of this are the gain scheduling control techniques that adapt feedback control gains according to a set of slow scheduling variables [3].

One limitation of this type of approach is that by defining a discrete set of operational points identification must be performed with fixed parameters. However, some types of rotating machinery, such as brake systems, present some parametric variations tied to wheel angular position [4]. It has also shown that the frequency response in a static brake is significantly different between the static and sliding conditions [4]. Thus a different way of characterizing systems with continuous parametric variations is needed.

This work proposes a strategy to identify modal properties in the presence of slow but continuous parameter variation. By interpolation between two tests where parameter variations are linked with a variable frequency harmonic excitation to approximately track phase resonance, the method gives estimates of natural frequency, damping and shape dependence on parameter variations.

Section 2 discusses the single degree of freedom parameter varying case. Extension of the practical multiple degree of freedom, limited observation, configuration is discussed 3. Finally section 4 illustrates application to a real test case.

2 Phase resonance, normal modes (1DoF example)

Let us in a first moment consider a one degree of freedom linear parameter-varying (LPV) system with natural frequency ω_n and damping coefficient ζ dependent on an external parameter p. The objective of this work is to estimate the parametric evolution of these two modal properties. Assuming that the time variation of p is slow enough to consider the system LPV, the transfer function from force to displacement is given by

$$H(\omega, p) = \frac{X}{F}(\omega, p) = \frac{1}{-\omega^2 + 2i\zeta(p)\omega_n(p)\omega + \omega_n^2(p)} = |H(\omega, p)|e^{i\varphi(\omega,p)} \tag{1}$$

In particular, this implies that the phase between displacement and force is a function of the excitation frequency ω and external parameter p

$$\varphi(\omega, p) = phase\,(H(\omega, p)) = atan2\left(-2\zeta(p)\omega_n(p)\omega,\ \omega_n^2(p) - \omega^2\right) \tag{2}$$

Phase resonance in the case of 1DoF linear time invariant systems is defined as the frequency for which phase φ equals -90^o between displacement and excitation force or a zero phase between velocity. Amplitude resonance coincides with phase resonance for velocity measurements and is slightly different for displacement and acceleration. The concept of phase resonance, initially introduced in force appropriation techniques, was further generalized by [5] to the case of non-linear systems, and combined in experiment with feedback techniques such as the phase-locked-loop [6, 7, 8] in order to extract modal properties as a function of vibration amplitude.

This work seeks to track phase resonance of a parametric system. An initialization phase based on broadband testing, such as the one in section 4, is used to generate a first rough approximation $\omega_{n0}(p)$ that is reasonably close to phase resonance. Performing a continuous parameter/frequency sweep, where p and the excitation frequency and linked by that approximate law $\omega_{n0}(p)$, leads for each value of p to a measurement point $H(p(t), \omega_{n0}(p(t))$ whose phase is marked by a star in figure 1. A second parametric experiment considers excitation at $\omega_{n0}(p) + \delta\omega_0$ also marked in the figure. If these two points are reasonably close to the phase resonance, a first order development of the phase gives

$$\left(\varphi(p, \omega) - \frac{\pi}{2}\right) = -\frac{1}{\omega_n(p)\zeta(p)}(\omega - \omega_n(p)) \ + \ O((\omega - \omega_n(p))^2) \tag{3}$$

where one can readily see that the intersection with -90^o gives the phase resonance and the slope the damping, in other words the updated estimates $\omega_{n1}(p), \zeta_1(p)$.

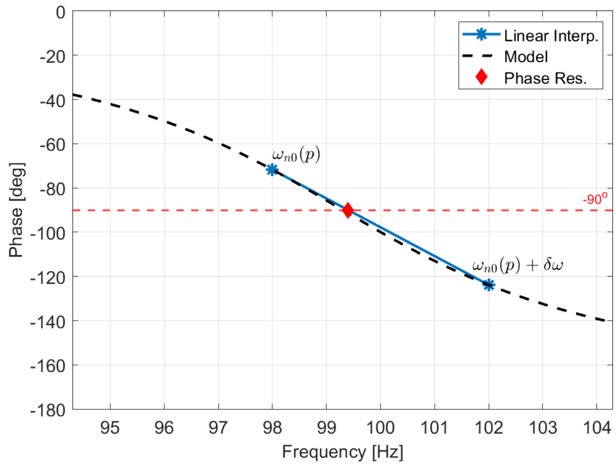

Figure 1: Example of the phase resonance estimation from the linear interpolation of phase for two neighboring frequencies.

This is very comparable to the appropriation and PLL approaches with the major difference that an iterative feed-forward strategy is used instead of feed-back. The advantage sought proceeding that way is that the process is inherently stable and leading to testing times known in advance.

Figure 2 illustrates a sample variation of natural frequency and damping ratio on parameter p. This evolution was created by interpolating between a set of four arbitrarily chosen key points.

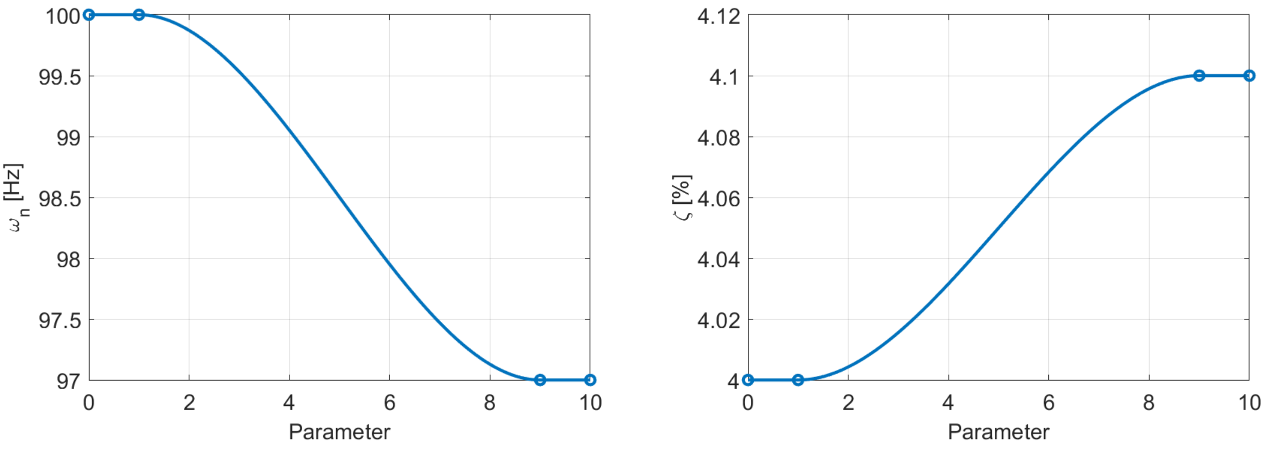

Figure 2: Parametric evolution of natural frequency (Left) and damping (Right)

Three test profiles linking p and excitation frequency $\omega_{n0}(p)$ are shown in figure 2 left. The first profile is the true $\omega_n(p)$ (blue), while the other two are respectively piece-wise cubic and linear interpolations of the key-points from figure 2 (red and yellow). The offset profiles are also displayed in figure 2 left for $\delta\omega = -1Hz$. The resulting phase corresponding to each input profile is obtained directly from equation (2) and show in figure 2 right.

At first glance one can observe that phase obtained for the approximate profiles show some level of oscillation that is not present in the exact tracking, showcasing the idea behind this phase resonance estimation method. It is important to note however that while the exact profile without offset yields a constant phase lag of -90^o the offset one presents a small level of variation due to the also changing damping coefficient.

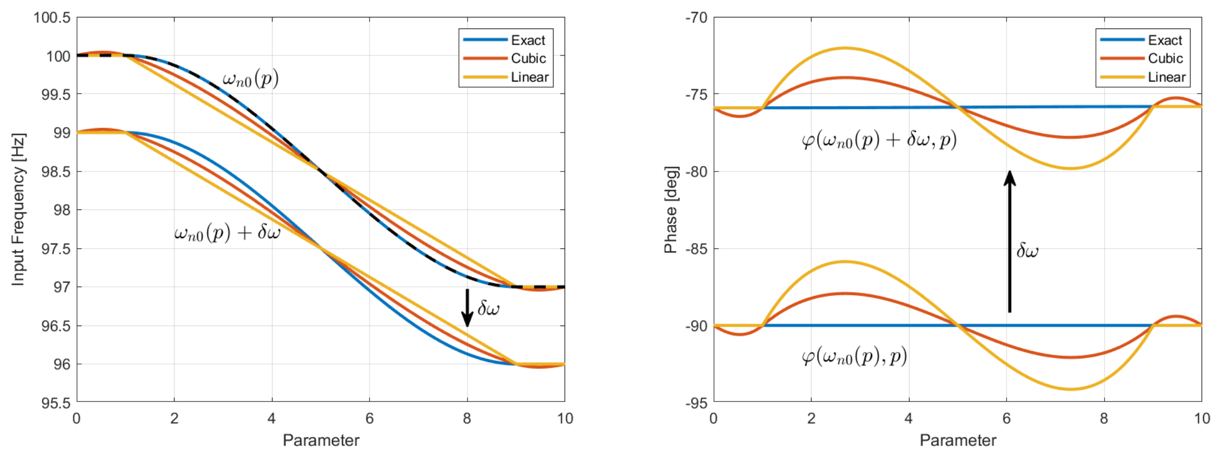

Figure 3: Input frequency (Left) and response phase lag (Right) as a function of external parameter for a 1DoF parametric system. Excitation frequency profiles: identical to phase resonance (Blue), piece-wise cubic (Red), and Linear (Yellow).

From the linear regression in (3), the values of $\omega_{n1}(p)$ and $\zeta_1(p)$ are then estimated. Figure 2 shows that while the natural frequency estimations superpose with the model values, damping has a tendency to be overestimated as a limitation of the first order approximation (3). In this example a $1Hz$ frequency offset resulted in a 1.5% damping bias, by reducing the offset to $0.5Hz$ the bias would be of 0.4%. Both of which are quite sufficient for industrial applications were bias of up to 20% in damping would be considered acceptable.

In order to improve, this estimation one could try to have both frequency/phase points closer to the $-\frac{\pi}{2}$ lag. Alternatively by increasing the number of frequency offsets $\delta\omega_i$ and then obtain for each value of p a number of points $[\cdots(\omega_{n0}(p)+\delta f_i, \varphi(\omega_{n0}(p)+\delta f_i, p))\cdots]$ allows for a better estimation of $\omega_n(p)$ and $\zeta(p)$.

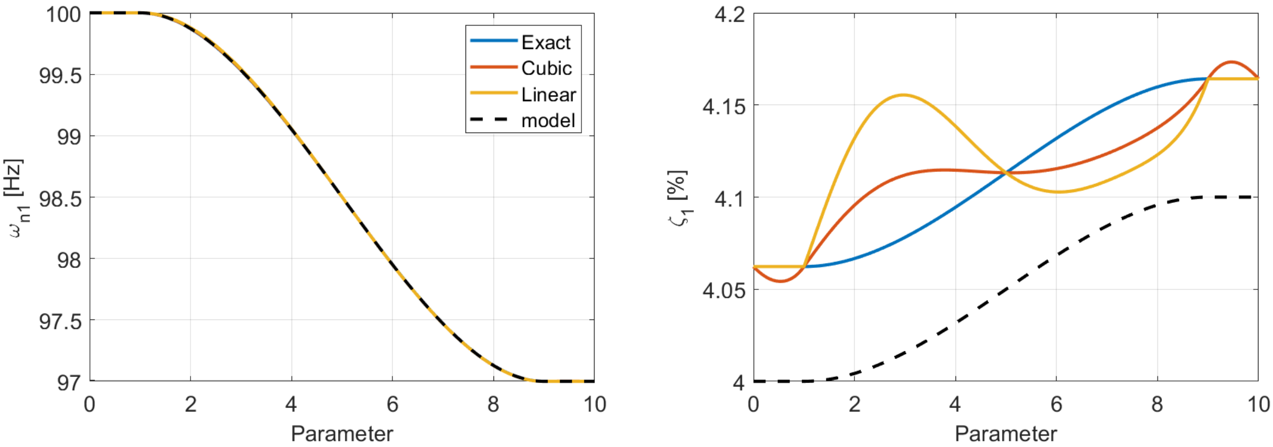

Figure 4: Re-estimation of phase resonance frequency and damping coefficient in a 1DoF system

3 Phase resonance in multiple degrees of freedom systems

In the case of systems with multiple degrees of freedom the definition of phase resonance requires some additional thought since multiple modes are seen in the response and non-proportional damping can lead to complex modes [9]. The notion of phase resonance is then classically defined for force appropriation [10, 11] or the non-linear normal mode methodologies [5, 6].

One possibility of how to define phase resonance in a multiple degree of freedom system is to use the concept of modal coordinates.

Let us consider the following multiple degree of freedom system excited by a force $[b]\{u\}$ with a set of measurements $\{y\}$

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=[b]\{u\} \qquad \{y\}=[c]\{q\} \tag{4}$$

Under the hypothesis of modal damping it is possible to write the transfer function of this system as the superposition of the response of all N_m modes of the system. Which implies that the response can be described in terms of a set of generalized amplitudes called modal coordinates that represent the amplitude of the structure vibration associated to a given mode shape [12, 9].

For simplicity we shall consider that mode shapes are mass normalized.

$$[H(s)]=[c]\left[[M]s^2+[C]s+[K]\right]^{-1}[b]=\sum_{j=1}^{N_m}\frac{[c]\{\phi_j\}\{\phi_j\}^T[b]}{s^2+2\zeta_j\omega_j s+\omega_j^2} \tag{5}$$

$$\{q(t)\}=\sum_{j=1}^{N_m}\{\phi_j\}x_j(t) \tag{6}$$

Using the mass orthogonality property of the mode shapes of a mechanical system it is possible to create a linear combination of the different degrees of freedom to obtain the modal coordinates creating a modal filter [13]. This concept allows one to use modal coordinates in an experimental setting, approaching the same ideas that are used in simulation [14]. Modal filtering has shown its usefulness in a number of different applications such as mode selection in autoresonance feedback excitation [7] and nonlinear modal testing [8].

$$[\Phi]=\left[\cdots\{\phi_j\}\cdots\right] \qquad \left([\Phi]^T[M]\right)[\Phi]=[I] \qquad \{x(t)\}=\left[\Phi^T M\right]\{q\} \tag{7}$$

One may however notice at this point that this formulation of a perfect modal filter considers that all degrees of freedom are measured without taking into account the observation at a finite number of sensors. Implying that some adaptations need to be made in order to extract modal coordinates from the observed mode shapes $[c\Phi]$.

$$\{y(t)\} = [c]\{q(t)\} = \sum_{j=1}^{N_j} \{c\phi_j\} x_j(t) \tag{8}$$

Note that while mode shapes $[\Phi]$ are mass orthogonal, the observed mode shapes $[c\Phi]$ may not be. So in order to estimate the modal amplitudes it is necessary to build a modal observation matrix $[c\Phi]^+$ such that $[c\Phi]^+[c\Phi] \approx [I]$ using a pseudo-inverse. Each of the lines in $[c\Phi]^+$ being a linear combination of the measurements $\{y\}$ that estimates a modal coordinate.

$$\{x(t)\} \approx [c\Phi]^+ \{y(t)\} \tag{9}$$

A good sensor placement is essential in order to allow the observation of all modal coordinates, for a further discussion on the subject refer to [15, 16].

In the absence of good identification, for example when noise or non-linearity is high enough to prevent identification of a normal mode model, modal filter approximations may still be needed. Using snapshots corresponding to measured frequency responses $[H(\omega_{Learn})]_{N_S\times(2N_T)}$ at a set of N_T frequencies ω_{Learn} loosely located close to one or more measured resonance, it well known that the response will be largely dominated by mode shapes. One can thus a singular value decomposition to create an ordered set of contributions to the response

$$\left[Re(\mathtt{H}_{Learn}) \quad Im(\mathtt{H}_{Learn})\right]_{N_S\times(2N_T)} = \sum_j \left\{u_j\right\}_{N_S\times 1} \left(\sigma_j \left\{v_{jc}^H \quad v_{js}^H\right\}\right)_{1\times 2N_T} = [u_j]_{N_S\times N_S} \left[\mathtt{q_{j1R}}\right]_{N_S\times N_T} \tag{10}$$

where the first of the N_S real left singular vectors $\left\{u_j\right\}$ correspond to mode shapes, the associated amplitudes at the learning points can be considered using the real imaginary split $\sigma_j \left\{v_{jc}^H \quad v_{js}^H\right\}$ or recombined into complex modal amplitudes $\left\{\mathtt{q_{j1R}}\right\}$. Note that rather than using FFT to obtain shapes, we currently work on an harmonic balance vector signal model [17] estimated using synchronous demodulation to properly account for slow time variations of system parameters.

A property of the singular value decomposition is that $[u_j]$ is an orthonormal matrix. The modal filter approximation is thus simply given by

$$\left\{\mathtt{q_{jR}}\right\} = [u_j]^T [\mathtt{q}] \tag{11}$$

where one will typically consider as many generalized coordinates as modes in the ω_{Learn} band, with this number being smaller than the number of sensors. The equivalence between using principal and true modal coordinates is not trivial, but this corresponds to a classical assumption of force appropriation [11] and the multivariate mode indicator function.

4 Application to a contact test bench

The proposed method is then applied to a test bench designed to evaluate the contact between piston and the backplate of a brake pad shown in Figure 4. This test bench is composed of a brake caliper that was cut in half and trimmed to expose the piston which presses the back-plate of a brake pad against a base. The caliper is held in place by two guide columns connected to the base. Pressure is applied via voltage controlled pressure generator.

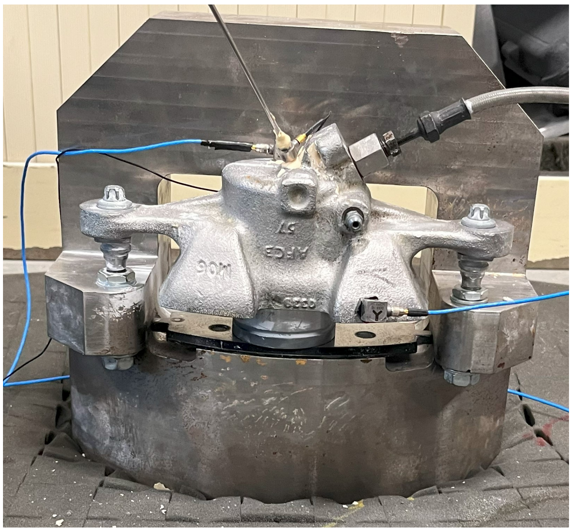

Figure 5: Piston/backplate contact test bench

Non-linearities affecting the dynamic behavior of this test bench come from the contact between the different parts, and the rubber seal that closes the pressure chamber and holds the piston in place inside the caliper. As a result the dynamic behavior is strongly dependent on the applied pressure, which during this experiment is considered as an external parameter. For the interpretation of results, the test bench is considered to be a linear parameter-varying system with the main parameter being pressure.

For this test, the bench was equipped with 2 tri-axial accelerometers (PCB piezotronics HT356B21) and excited via a electrodynamic shaker (Brüel&Kjaer vibration exciter 4809) connected to a power amplifier. The applied load was assessed using a load cell (Brüel&Kjaer NCEB018) and a conditioning amplifier. Measurements were made using a modular acquisition system (NI cDAQ-9174) with one analog output (NI 9263) and 8 analog inputs in two NI 9234 modules.

A first characterization of the parametric effect is obtained using standard experimental modal analysis techniques for fixed pressure values. More specifically a swept sine excitation between 100Hz and 3kHz was applied for 8 pressure values from 2 to 16 bar. This characterization resulted in a series of transfer functions that describe the systems evolution with pressure. We will focus on the first system mode whose evolution is shown in figure 6 right.

Some perturbations can be seen in the measured transfer functions, the two most notable ones being the interaction between the test bench and excitation system that can be seen by the variable level of effort applied (figure 6 left) and the harmonics of 50Hz, which can be seen in 6 right, suggesting a problem with the electrical ground connection.

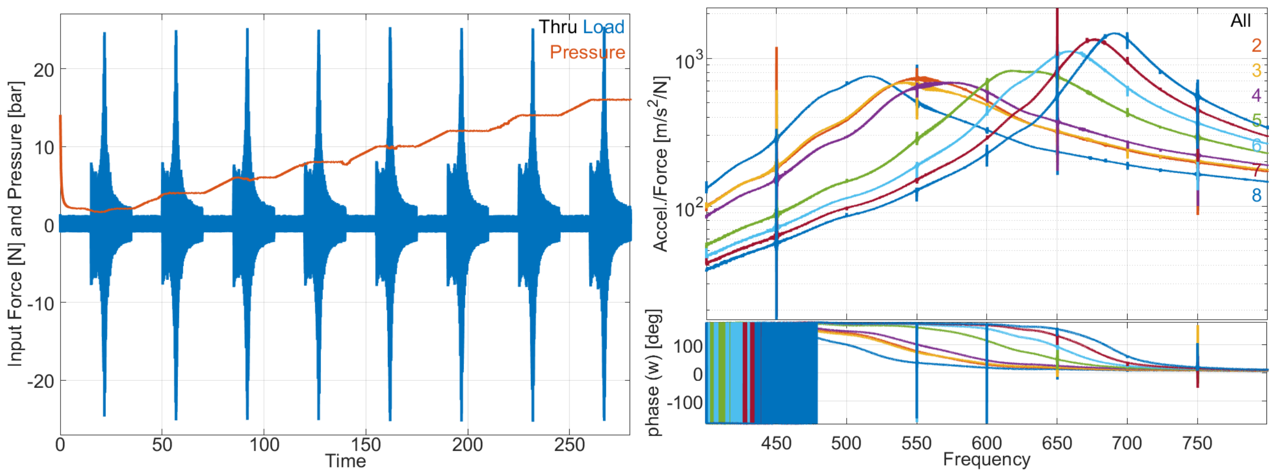

Figure 6: Left: pressure and input force applied to the test bench. Right: transfer functions corresponding to different pressure levels (Right)

A mode identification [18] allows for a first estimation of the resonance frequency and damping coefficient at the selected pressure values. A first estimate of the natural frequency dependence on pressure $\omega_{j0}(p)$ is then built using the simple linear interpolation shown in figure 7.

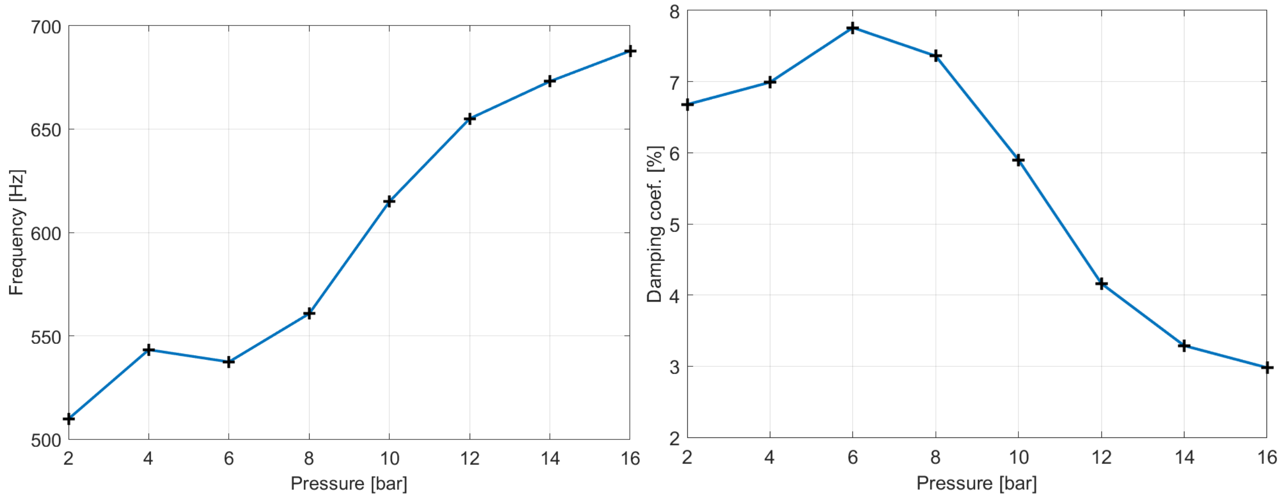

Figure 7: Identified natural frequency (Left) and damping coefficient (Right) evolution with pressure using swept sine excitation at fixed pressure.

Next a slow triangular pressure profile is defined and a corresponding frequency profile is constructed using $\omega_{j0}(p)$ both of which are shown in figure 8. This simultaneous evolution is then applied to the test bench with two different offset values 0 and $+10Hz$. One can notice a pressure transient in the first 20s of the measurement without offset as it converges to the input profile. Also, irregular drops in pressure can be seen in the descent phase suggesting some stick-slip occurrences on the pressure generator.

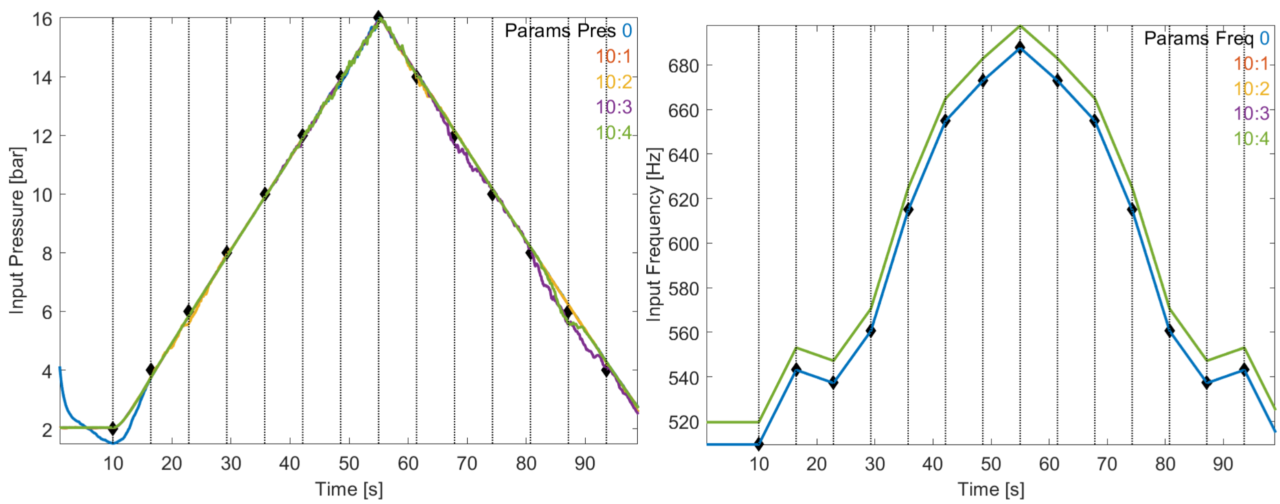

Figure 8: Pressure and frequency profiles used in the parametric sine tests

Next, using a synchronous demodulation based strategy [17], the complex amplitude associated to each sensor can be estimated. From which the first generalized amplitude, shown in 9 right, can be extracted. One can easily note on figure 9 left that the same overall trends can be observed the sensors with higher amplitude. This fact shows itself on the fact that all measurements were dominated by a single principal component, allowing us to represent the vibration by a single generalized amplitude.

In order to ensure that all measurements are being projected into the same set of generalized coordinates, one measurement was arbitrarily chosen as reference and the principal shapes were extracted using singular value decomposition (10). The other measurements were then projected onto the same principal shapes using (11). It is noticeable on figure 9 right that the extracted generalized amplitude in the four measurements with a $+10Hz$ is consistently repeatable presenting only some level of error in he zones were pressure fluctuations are seen in figure 4. It is also very clear that for the measurement without offset the first 20s present large phase variations which can be associated to the pressure drop that is not present in the other measurements.

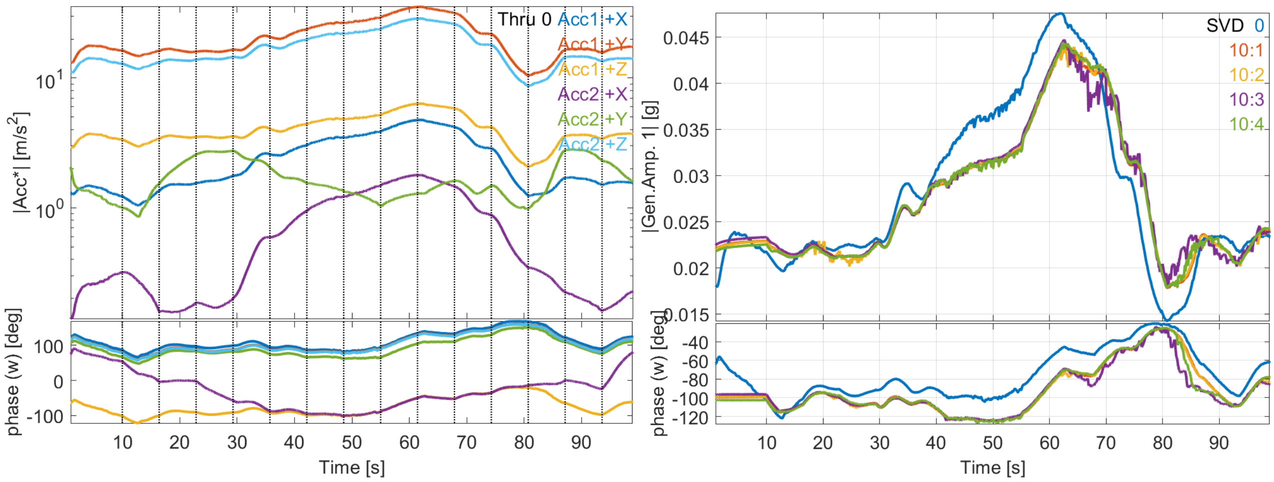

Figure 9: Left: complex amplitude of the first harmonic in different accelerometers in a single measurement. Right: generalized coordinate associated with the first principal shape for different measurements with and without frequency offset

From the measured phase difference phase resonance frequency and damping can be estimated using the procedure described in section 2. For the the first 20s shown in 10 left, the estimated values are not very good as the applied pressure was not yet stable in the measurement without offset (figure 8 left). From 20 to 55s, the estimated natural frequency follows the same overall trend as the original estimated profile. An interesting behavior can then be observed in the pressure descent phase after 55s, natural frequency remains constant for a while before starting to decrease. This suggests some level of hysteresis in the pressure dependence and motivates the proposed idea of iterating on the estimates of this dependence. Finally, between 70 and 80s some level of fluctuation in the estimated parameters can be seen as the phase values are further away from -90^o and thus the linear approximation of the frequency/phase relation in no longer ideal.

When frequency is not properly estimated, between 0s and 20s and between 70s and 80s, damping estimates are quite different and thus probably biased (figure 10). In other regions, damping values are found to be much lower than those extracted with large band signals dropping from values around 5% in figure7 to values around 0.2% in figure 10. It clear that both tests have notably different variations of the excitation levels with time (mostly constant in the sine test, variable in the sweep as shown in figure 7). It is thus expected that the contact state averaged over a period may differ significantly, and this can generate apparent damping [19]. Our expectation is that measuring higher harmonics [20] will provide insight.

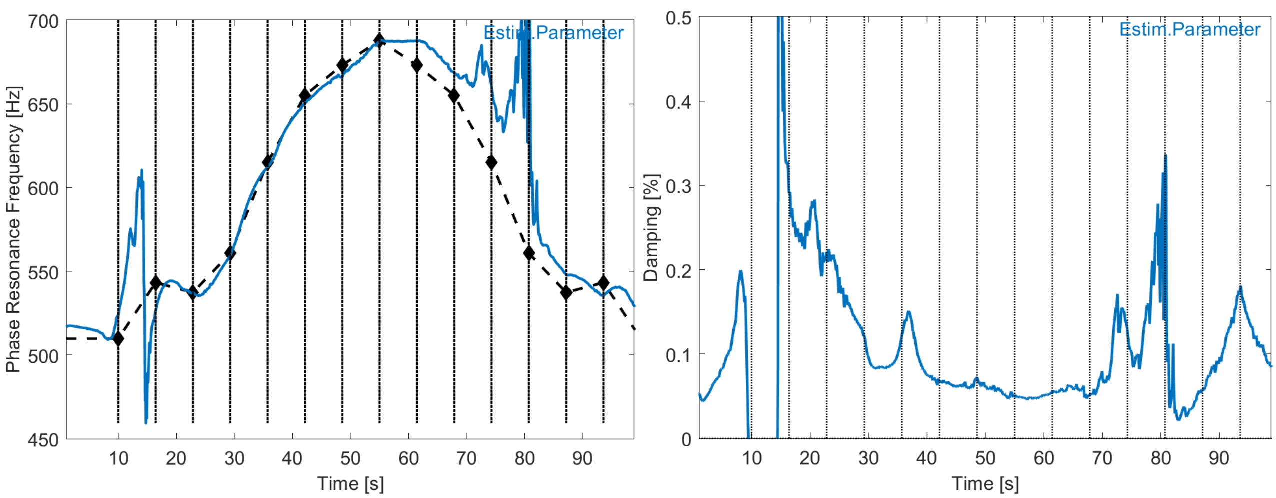

Figure 10: Re-estimated natural frequency and damping for different sensors, vertical lines indicate the crossing of a learning pressure value.

5 Conclusion

In conclusion, this work has proposed an experimental method to identify the parametric evolution of natural frequency and damping coefficient in the presence of slow but continuous parameter variations. The proposed method is a iterative feed-forward approach and thus allows experiments that are guaranteed to give a result in a have a controlled time.

The results are quite encouraging and demonstrate how the proposed strategy gives a good understanding of parametric dependence and limitations in the parameter control of the bench. Future tests, will consider multiple iterations of the process, as this is expected to be necessary to separate trends linked to bench/procedure limitations from non-linear trends of the bench. Analysis of higher harmonics is also implemented and expected to given a useful characterization of non-linearities within the harmonic balance framework [17].

Acknowledgments

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