Identification of the Payne effect in a viscoelastic material coupling Bayesian identification and Digital Twin

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Abstract

The Payne or Fletcher-Gent effect is a particular behavior occurring in viscoelastic materials containing fillers. It induces a nonlinear dependency of the viscoelastic storage modulus on the amplitude of the applied strain. This amplitude dependency must be taken into account in engineering applications as it does change the elasto-dynamic behavior of the overall structure integrating the material. A methodology is developed in the proposed work to identify this nonlinear effect. The approach first starts with the identification by Bayesian inference of the frequency and damping properties of a viscoelastic sample. The input data are obtained from a modified Oberst test, conducted for different displacement load amplitudes. A digital twin of the experimental set-up is then used to evaluate the stiffness behavior, and in particular the Young's modulus of the material, at the measured frequencies, and to deduce the strain level within the sample. This work shows that the Payne effect, which leads to the decrease with the amplitude of the Young's modulus, can thus be estimated in terms of mean and standard deviation by combining experiments and numerical simulations.

1 Introduction

Viscoelastic materials are widely used for their stiffness and damping properties in vibration control solutions. Their behaviour can be complex, depending on frequency or temperature, or non-linear. The Payne effect [1][2], which occurs in particular within rubber-filled materials, leads to a monotonous decrease in storage modulus with the increasing dynamic strain amplitude. In order to take into account this phenomenon in a design process, representative models are required, and only few works have been carried out to identify this. Experimental studies mainly consists of dynamic mechanical analyses (DMA) [4] [3], which have the disadvantage of being limited in terms of strain levels, or of being costly in terms of implementation time. A methodology to identify the Payne effect is presented in this work, based on the Oberst-beam technique :

2 Identification process

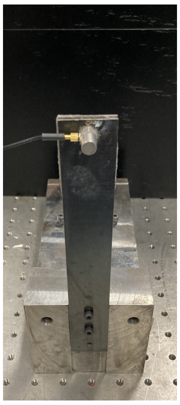
The identification process combines the stochastic post-processing of data obtained with the Oberst Beam Method, and results coming from a digital twin of the test, to obtain the evolution of the storage modulus as a function of the strain level.

2.1 Experimental set-up

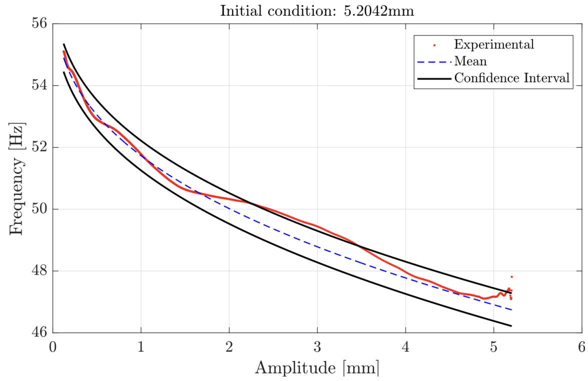
The Oberst set-up consists in a beam composed of two stainless steel plates surrounding the silicone sample to characterize (Figure 1a). Different initial conditions are applied to the free end and the free vibratory response is recorded : the quantity of interest is the nonlinear evolution of the natural frequency associated with the first bending mode. Bayesian calibration [5] for the frequency/amplitude and damping/amplitude backbones is performed used the set of free response data to update a model define in a prior condition : this procedure allows to built a statistical model with confidence intervals. As an illustration, Figure 1b presents the model obtained for the frequency/amplitude backbone with an experimental result for an initial displacement of 5.2 mm: the corresponding equation is written as,

$$\omega_0^2 = \omega_{n_0}^2 + \delta_f \mathscr{Q}^{1/3} \tag{1}$$

where $\omega_{n_0} = 363.3\,\mathrm{rad/s}$ and $\delta_f = -2.6457 \times 10^5\,\mathrm{N/(kg\,m^3)}$. $\mathscr{Q}$ denotes the amplitude of the displacement. This plot allows to define the experimental frequency band of interest which represents the frequency band of modulus variations.



(a) Oberst Beam Set-up : an initial displacement is applied at the free end.



(b) Experimental Backbone : Mean and Confidence Interval obtained with Bayesian calibration, Experimental curve for an initial displacement of 5.2 mm.

Figure 1: Experimental set-up : measurements are done on an Oberst beam set-up (a) and used for Bayesian calibration of the frequency/amplitude backbone (b).

2.2 Digital twin

A digital twin of the experimental set-up (Figure 2a) is developed, firstly to assess the evolution of the Young modulus depending on the frequency and secondly, to define a ratio between the shear strains and the displacements undergone. A parametric analysis is done to compute the first natural frequency for different values of the Young modulus, and thus determine the evolution of the Young modulus as a function of the frequency (Figure 2b). On the one hand, from the knowledge gathered on the experimental frequency band of interest it is possible to determine the equivalent experimental frequency evolution for the Young modulus. On the other hand, from the knowledge of the experimental displacement, the experimental strain inside the specimen was inferred.

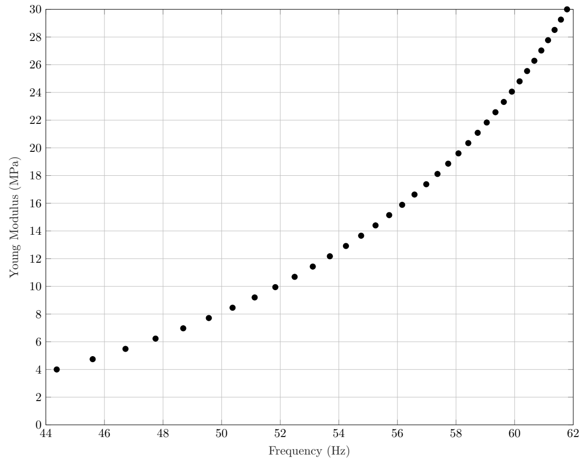
2.3 Synthesis

Finally, the evolutions of the Young modulus experimentally identified as function of the strain inferred by the Digital Twin are combined to evaluate the Payne effect (Figure 3). As expected, this effect is manifested through nonlinear dependency between the storage modulus and the shear strain amplitude. This phenomena may impact the elasticity, and thus the rigidity, of a structure and has to be taken into account during designing simulations.

3 Conclusion

A methodology has been developed to identify the Payne's effect inside a viscoelastic material. It combines experimental measurement of free vibratory responses on an Oberst Beam, bayesian identification from these mesurements of frequency/amplitude and damping/amplitude backbones, and finally combination to a digital twin to estimate experimental strains. The Bayesian inference approach offers the benefit of estimating the confidence interval : the case studied leads to a small interval, which demonstrates the promising and reliable nature of the approach developed to estimate this effect experimentally.

(a) Digital Twin of the Oberst Beam Set-up



(b) Evolution of the Young Modulus depending on the frequency determined from parametric numerical simulations.

Figure 2: Digital twin : a digital twin of the Oberst beam set-up is built (a) and a parametric analysis is done to build the evolution of the Young modulus depending on the frequency (b).

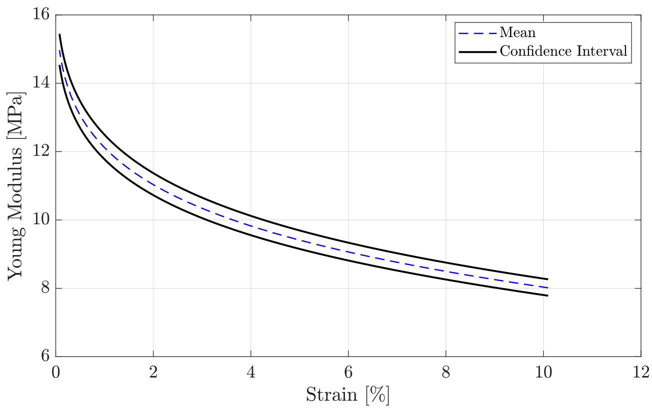


Figure 3: Evolution of the Young Modulus depending on the Strain.

Acknowledgements - additional information

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