

Understanding vibroimpact damping through a numerical energy based approach

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Abstract

Vibroimpact damping classically consists in the integration in a main vibrating structure of oscillating masses. The energy is dissipated thanks to the impact interactions with the main vibrating structure. An assumption is that energy is transferred for the low excited modes towards the higher modes of the structure, thanks to nonlinear effects. This leads to a faster and more efficient energy dissipation. This hypothesis is investigated within the framework of the work presented, in the case of a beam equipped with a vibroimpact absorber, assuming conservative impacts. A numerical study is developed to solve the equations of motion and derive energy balance with negligible error. Then, the energy transfers are assessed in the case of a harmonic excitation centered on not only the first, but also the second mode, demonstrating the effectiveness of the vibro-impact absorber in each case. Additionally, the way the energy is dissipated is explained and quantitatively assessed. At least, the results obtained for the first modes are compared.

1 Introduction

Vibroimpact absorbers are recognized for their ability to attenuate undesired structure vibrations. They consist in small masses constrained to oscillate between fixed boundaries inside a main structure, in order to generate energy loss during collisions. The efficiency has been demonstrated for free [1], harmonic [2] as for random excitations [3]. The effectiveness depends on the clearance, on the restitution coefficient at the contact interface, and on the mass ratio between the oscillating mass and the main structure [4]. Damping is linked to an energy transfer from low to high modal frequencies that occurs during impacts. This intermodal energy transfer has been demonstrated experimentally in [5] by using projections in the modal space coming from a numerical model, and maximized in [6] in the least possible time working on the clearance distribution. The purpose of the work presented is to analyze this energy balance in the case of a beam including vibroimpactors under a harmonic excitation as investigated in [7], not only for the case of the first mode but also for the second one. A numerical energy-based approach is used for that purpose.

2 Dynamic study of a beam with vibroimpacts

The study case is a cantilever beam submitted to a bending load with an vibroimpact absorber attached to its free end (Figure 1). The equation governing the dynamic of the beam is:

$$E.I. \frac{\partial^4 w}{\partial x^4} + \rho.A. \frac{\partial^2 w}{\partial t^2} = \delta(x - x_{exc}).F_{exc}(t) + \delta(x - x_{VI}).F_{VI}(u_b, w); \quad (1)$$

while the second Newton's law applied to the ball gives:

$$m_b.\ddot{u}_b = -F_{VI}(u_b, w). \quad (2)$$

where F_{VI} is the contact force of the ball on the beam during impacts. Parameters are defined in Appendix A. As the purpose is to study the energy distribution over modes, a modal truncation to a finite number of modes N fixed to eight is applied such that,

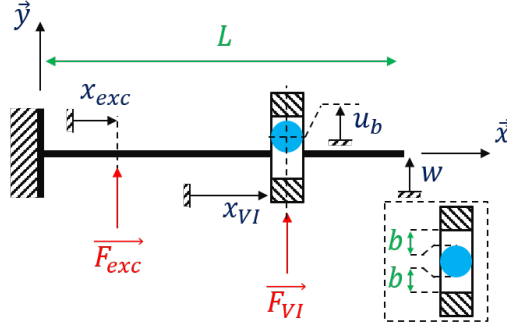


Figure 1: Clamped-free beam with an attached vibro-impact absorber.

$$w(x, t) = \sum_{i=1}^N \varphi_i(x) \cdot q_i(t). \quad (3)$$

Reinjecting Equation (3) in Equation (1), the generalized coordinates $\{q\}$ can be determined by solving the matrix system,

$$[M] \cdot \{\ddot{q}\} + [C] \cdot \{\dot{q}\} + [K] \cdot \{q\} = F_{exc} \cdot \{\varphi(x_{exc})\} + F_{VI} \cdot \{\varphi(x_{VI})\}; \quad (4)$$

where $[M]$, $[C]$, $[K]$ denote the modal mass, damping and stiffness matrices. If unitary modal masses are used, then $[M] = [\mathbb{1}]$. The impact is described using a restitution coefficient e . As the purpose is to derive exact energy balance, only analytical formulation are used for the mode shapes in particular. Finally the coefficient of restitution is fixed to one to investigate the energy transfer even in the case of non-dissipative impacts.

3 Energy-based approach

The behavior of the beam is studied under harmonic excitation. The energy-based approach relies on the following energy quantities, considering the system beam+ball submitted to the external excitation. For the i -th mode of the beam, are defined:

- The kinetic energy $E_{kin}^i = \frac{1}{2} \cdot m_i \cdot \dot{q}_i^2$;
- The potential energy $E_{pot}^i = \frac{1}{2} \cdot k_i \cdot q_i^2$;
- The mechanical energy $E_{mec}^i = E_{kin}^i + E_{pot}^i$;
- The dissipated energy $E_{diss}^i = \int_0^t c_i \cdot \dot{q}_i^2 \cdot dt$;
- The energy of the excitation $E_{exc}^i = \int_0^t F_{exc}(t) \cdot \varphi_i(x_{exc}) \cdot \dot{q}_i(t) \cdot dt$;
- The kinetic energy of the ball $E_{ball} = \frac{1}{2} \cdot m_b \cdot \dot{u}_b^2$.

In order to compute the previous terms without approximations, the dissipated energy E_{diss}^i and the energy of the excitation E_{exc}^i which are expressed with an integral are calculated analytically instead of using numerical integration.

4 Harmonic excitation on the first and on the second mode

Let first consider the case of an excitation frequency around the first bending mode. Figure 2 depicts the average variation over multiple impacts of the mechanical energy ΔE_{mec}^i for the i -th mode normalized by the energy of the entire system $\{\text{beam} + \text{ball}\}$ at the moment of the impact $E_{syst} = \sum_{i=1}^N E_{mec}^i + E_{ball}$, so that the figure represents the quantity $\frac{\Delta E_{mec}^i}{E_{syst}}$ for each mode. These results show that the first mode loses almost 10% of the total energy, while the modes 2 to 8 receive around 1,5% of the total energy. Between the impacts, the

mechanical energy on each mode is dissipated. And the higher the mode, the faster the energy is dissipated. It is also interesting to note the equipartition of the energy on the higher modes, with an assumption of a purely conservative contact behavior.

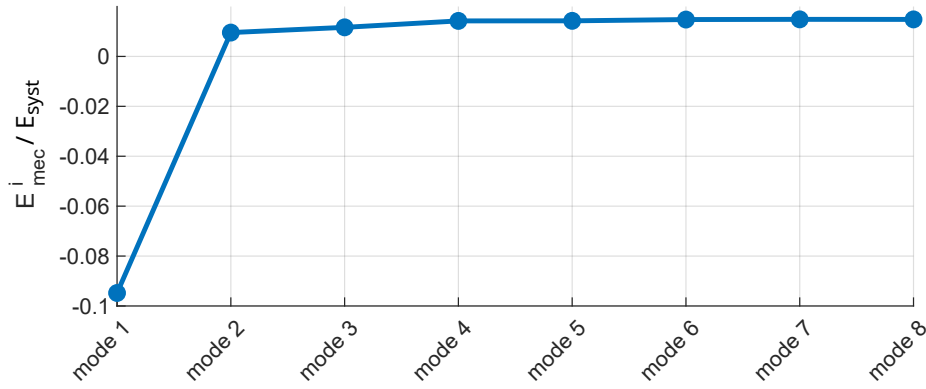


Figure 2: Average variation of energy during the impacts.

Figure 3 now presents the mechanical energy distribution for an excitation frequency near the second mode, for different clearance configurations (A: 0.121 mm; B: 0.969 mm; C: 1.21 mm; D: 1.82 mm) such that C is the more efficient one. The energy transfer is maximized from the excited mode towards the higher mode. It is worthy to note that a negligible amount of energy is transferred to the lower modes (-0,3 % for the first mode).

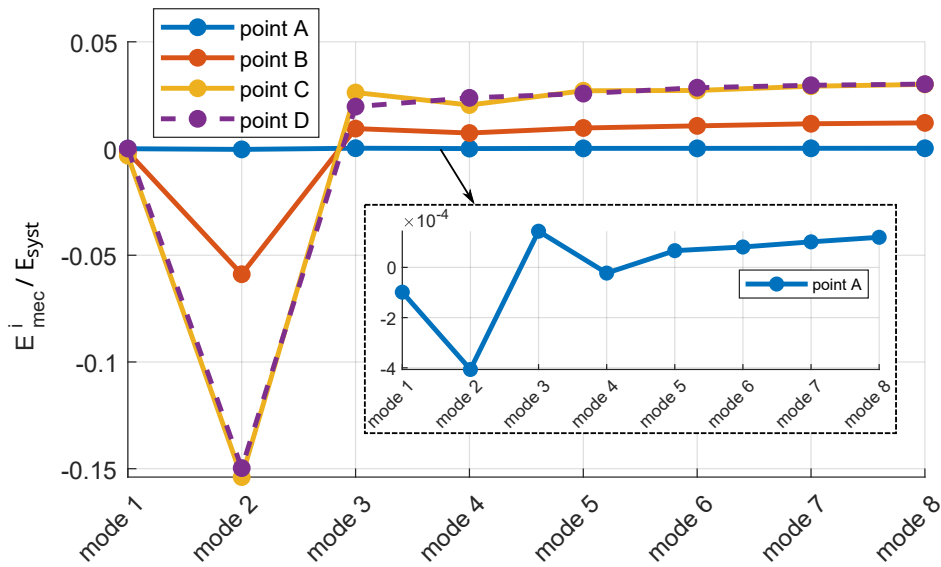


Figure 3: Energy received by mode for an excitation on the second mode.

5 Conclusion

The work focuses on the energy transfers due to vibroimpacts into a beam. Analyses are done on the average energy transferred to the modes in proportion of the mechanical energy available in the system throughout the impacts. In the case of an excitation on the first mode, the results highlight an energy transfer from the first excited mode towards the higher modes of the beam. Moreover, considering the modes four to eight, the energy is uniformly distributed between them. Two beneficial phenomena are identified. Firstly, the higher modes have a lower amplitude of vibration. Thus, the less energy is contained on the lower modes, the smaller the amplitude of vibrations. Secondly, as the higher modes realize more oscillations in the same amount of time than the lower modes, the energy dissipation occurs faster and the vibrational energy is dissipated more efficiently. Besides, the case of an excitation on the second mode reveals that the energy transfers towards the lower modes are

negligible. Designing a vibroimpactor that optimises energy transfer to high-frequency modes would therefore appear to be a relevant strategy for maximising vibration reduction performance

Acknowledgements - additional information

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A Appendix

The following list defines the parameters of the problem:

- $w(x,t)$ is the transverse displacement of the beam;
- $u_b(t)$ is the displacement of the ball;
- x_{exc} is the position of the external excitation force;
- x_{VI} is the position of the vibro-impact absorber on the beam;
- L is the length of the beam;
- b is the half-gap of the cavity (the ball can oscillate over a distance equals to $2.b$);
- m_b is the mass of the ball;
- ρ is the density of the material of the beam;
- E is the Young's modulus of the material of the beam;
- I is the second moment of area of the beam;

- A is the area of the cross-section of the beam;
- $\overrightarrow{F_{exc}}$ is the external exciting force;
- $\overrightarrow{F_{VI}}$ is the contact force of the ball on the beam during impacts.