Robust estimators of autocorrelation function in application to local damage detection for non-Gaussian signals

Wojciech ŻUŁAWIŃSKI[1], Jérôme ANTONI[2]
Radosław ZIMROZ[1], Agnieszka WYŁOMAŃSKA[1]

[1]Wroclaw University of Science and Technology, Wybrzeze Wyspianskiego 27
50-370 Wroclaw, Poland
[2]Univ Lyon, INSA-Lyon, LVA, EA677, F-69621 Villeurbanne, France
wojciech.zulawinski@pwr.edu.pl

Abstract
In this extended abstract, we briefly present the application of robust estimators of the autocorrelation function (ACF) in cyclostationarity detection methods which may be used for local damage detection. Such approach is devoted for signals with heavy-tailed, non-Gaussian behaviour, for which standard methods might fail. We consider the modified cyclostationarity detection algorithms, where the classical ACF estimator is replaced with a robust one. The results obtained for a simulated signal show the advantage of the proposed approach.

1 Introduction

One of the most common approaches for local damage detection is the cyclostationary analysis. The indicators of cyclostationarity are based on the classical estimation of the autocorrelation function (ACF), called sample ACF. It can be applied for the underlying signal in time, time-frequency or frequency-frequency domains. The indicators based on sample ACF are very efficient in case when the informative signal is disturbed by Gaussian- (or close to Gaussian)-distributed noise. However, in case when the background noise has strong non-Gaussian behavior, the sample ACF may fail as it is sensitive to large impulses related to the non-Gaussian characteristics of the noise. Thus, in this work, we discuss the new approach based on the robust versions of sample ACF. By robust sample ACF we mean the algorithms less sensitive to large observations that estimate theoretical ACF. By relatively simple replacement of the classical statistic by its robust versions, one may decrease the influence the non-Gaussian distribution and identify the cyclostationary behavior also in this case. In the literature, there are considered various statistics used as robust versions of sample ACF but they were never used in condition monitoring. In this extended abstract, we demonstrate the general methodology of new cyclostationary indicators that, similar as the classical approach, can give the information in different domains. We present the comparison of three selected robust estimators of ACF using the simulated signal with the α-stable distribution of the background noise.

2 Methodology

The standard estimator of correlation (in the Pearson sense), called sample correlation, for zero-mean vectors $\mathbf{w}_1 = (w_1^1, w_1^2, \cdots, w_1^N)$ and $\mathbf{w}_2 = (w_2^1, w_2^2, \cdots, w_2^N)$ is defined as:

$$M_1(\mathbf{w}_1, \mathbf{w}_2) = \frac{\frac{1}{N}\sum_{i=1}^{N} w_1^i w_2^i}{std(\mathbf{w}_1)std(\mathbf{w}_2)}, \tag{1}$$

where $std(\mathbf{w}_k)$ is the sample standard deviation corresponding to the vector $\mathbf{w}_k$ for $k = 1, 2$. If $\mathbf{w}_1$ and $\mathbf{w}_2$ are realizations of the same process but in different time points, then the sample correlation is called sample autocorrelation function (sample ACF), while its non-normalized version (i.e. without the normalization with the sample standard deviations) — the sample autocovariance function (sample ACVF). As mentioned, the sample ACF is an efficient tool for Gaussian data, but tends to fail in presence of outliers. Hence, we consider the following robust correlation estimators. In the first one, called trimm, we start with constructing:

$$\mathbf{w}_3 = (w_3^1, w_3^2, \cdots, w_3^N) = (w_1^1 w_2^1, w_1^2 w_2^2, \cdots, w_1^N w_2^N),$$
$$\tilde{\mathbf{w}}_k = \{w_k^i : i = 1, \ldots, N;\ w_3^{(g)} < w_3^i < w_3^{(n-g+1)}\}, \quad k = 1, 2,$$

where $g = \lfloor c \cdot N \rfloor$ for a trimming constant $0 \leq c < 0.5$ (here $c = 0.025$), and $w_3^{(1)}, w_3^{(2)}, \ldots, w_3^{(N)}$ is an ordered vector $\mathbf{w}_3$ in ascending order. Finally, we calculate the trimm estimator:

$$M_2(\mathbf{w}_1, \mathbf{w}_2) = M_1(\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2). \tag{2}$$

Moreover, we consider the Kendall correlation defined as:

$$M_3(\mathbf{w}_1, \mathbf{w}_2) = \sin\left(\frac{\pi \rho_K(\mathbf{w}_1, \mathbf{w}_2)}{2}\right), \text{ where } \rho_K(\mathbf{w}_1, \mathbf{w}_2) = \frac{2}{N(N-1)} \sum_{i<j} \operatorname{sgn}((w_1^i - w_1^j)(w_2^i - w_2^j)), \tag{3}$$

and the Spearman correlation given by the formula:

$$M_4(\mathbf{w}_1, \mathbf{w}_2) = \sin\left(\frac{2\pi \rho_S(\mathbf{w}_1, \mathbf{w}_2)}{6}\right), \text{ where } \rho_S(\mathbf{w}_1, \mathbf{w}_2) = \frac{\frac{1}{N}\sum_{i=1}^N r_1^i r_2^i}{std(\mathbf{r}_1) std(\mathbf{r}_2)}, \tag{4}$$

for $\mathbf{r}_1$ and $\mathbf{r}_2$ being (zero-mean) rank vectors for $\mathbf{w}_1$ and $\mathbf{w}_2$, respectively. Although Kendall and Spearman correlations are often defined as just $\rho_K(\mathbf{w}_1, \mathbf{w}_2)$ and $\rho_S(\mathbf{w}_1, \mathbf{w}_2)$, respectively, with the transformations applied in (3) and (4) they can be considered as robust estimators of Pearson correlation [2].

In this work, we consider the application of robust ACF estimators to the cyclic spectral coherence (CSC). It is a classical method for cyclostationarity detection where the bi-frequency maps are constructed; however, in its original version it is sensitive to outliers. Hence, we modify it by replacing the sample ACVF with the proposed robust ACF estimators. To calculate CSC, we used the averaged cyclic periodogram algorithm [3].

3 Results

The presented methodology is applied to the simulated signal from the following model:

$$X_t = s(t) + Z_t, \tag{5}$$

where $s(t)$ is the deterministic component, called signal of interest (SOI). It is a series of cyclic impulses with frequency $f_f = 30$ Hz. A single impulse is defined as a decaying harmonic oscillation with amplitude $B = 45$ and carrier frequency $f_c = 5000$ Hz. Moreover, the background noise (random component) $\{Z_t\}$ is here a sequence of independent identically distributed (i.i.d.) random variables from symmetric α-stable distribution ($S\alpha S$) with $\alpha = 1.7$ and $\sigma = 3$. We consider the sampling frequency $f_s = 25000$ Hz and the sample consisting of 50000 observations (2 seconds). The simulated signal and its spectrogram are presented in Fig. 1. As one can see, the cyclic impulses are hidden behind the heavy-tailed background noise.

In Fig. 2, the spectral coherence maps based on the sample ACVF and considered robust ACF estimators are presented. Let us note that on the map for the classical estimator (top left) one cannot detect any periodicity. On the other hand, for all robust estimators the cyclic impulses are identifiable. The low efficiency of the classical method is directly caused by strong outlying non-periodic impulses present in the signal, related to the heavy-tailed non-Gaussian distribution of the background noise. As such a behaviour of a signal may occur in reality, due to specific processes conducted by the machine (e.g., cutting, crushing, drilling), it may be useful to consider the proposed robust methodology.

4 Conclusions

In this extended abstract, we demonstrate the part of our ongoing research, where we present the efficiency of the robust estimators of ACF applied to the non-Gaussian vibration signals for local damage detection. The universality of the proposed approach is demonstrated for various types of non-Gaussian distributed noises. The simulation results are supported by the applications to real signals from mining machine, see [1] for more details.

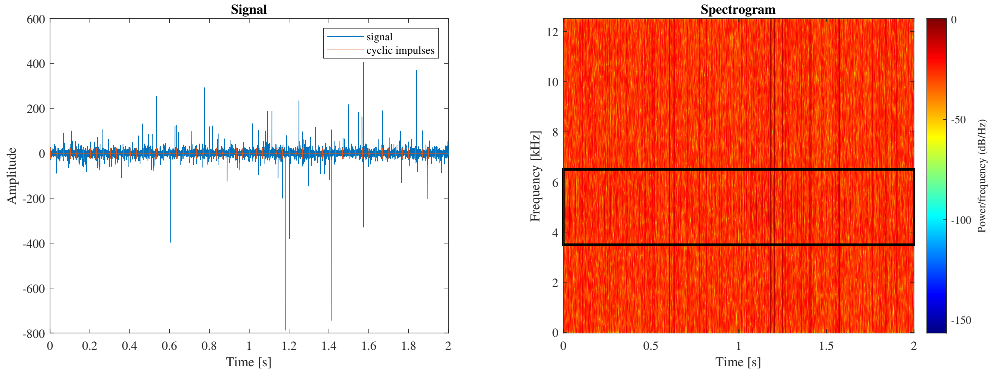

Figure 1: Simulated signal from the model $\{X_t\}$ for $\{Z_t\} \sim S\alpha S(\alpha = 1.7, \sigma = 3)$ and its spectrogram (with the true informative frequency band marked).

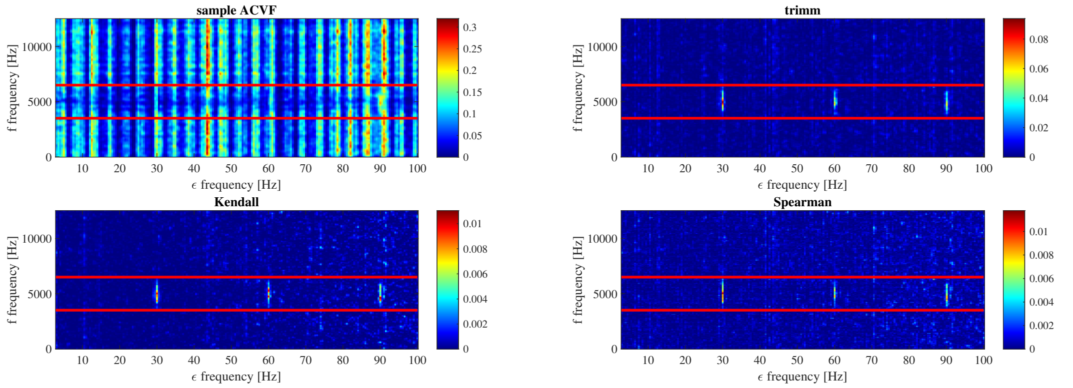

Figure 2: Spectral coherence maps for the simulated signal (with the true informative frequency band marked).

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