

Bearing diagnostics based on a Spectral combination of Hjorth's parameters

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Abstract

Signal decomposition method based on spectral criteria or more recently data-driven, are the basis of bearing and gear fault diagnosis. In this work, a new signal decomposition method based on a Spectral combination of Hjorth's parameter is proposed. The effectiveness and sensitivity of such a signal decomposition to bearing fault diagnostics has been investigated based on both synthetic and real bearing vibration signals. The results show good performances in extracting the frequency band which carries the most useful diagnosis information, and sensibility to the different fault severities.

1 Introduction

In the realm of gear and bearing diagnostics a plethora of condition indicators and signal decomposition has been introduced since 1980s, with the aim of extracting useful information concerning the machine state of health. Among the other, Sharma and Parey [1] proposed a review on several condition indicators for gear faults, whilst Miao et al. [2] used sparsity indexes for fault diagnosis of rating machinery. Lately, Antoni and Borghesani [3] proposed a statistical framework for designing condition indicators based on a generalized likelihood ratio. Moreover, Wang et al. [4] prove that spectral version of the previously mentioned indexes can be expressed as the sum of weighted normalized square envelope. The main difference among these metrics is that different weights are respectively applied to normalized square envelope. Nowadays, new emphasis has been given to signal decomposition methods that aims to return components which maximize a particular criterion or signal properties such as kurtosis [5], negative entropy [6] or cyclostationarity [7]. Usually, this information is given in a frequency/frequency resolution plane defining the well-known "kurtogram" [8] and "infogram" [6]. The kurtogram, which is a combination of Spectral Kurtosis for different frequency bands, can be seen as a signal decomposition into kurtic components. Such decomposition is a pivotal diagnosis tool for impulsive faults in rotating machines. However, this kurtic decomposition suffers in case of impulsive noise, which could drives the decomposition due to the highest kurtosis value. This shortcomings was fixed by Antoni in [6], inspired by the concepts of thermodynamics in which a transient phenomena could be seen as a departure from the system state of equilibrium. Therefore, by the analysis of the entropy of the squared envelope he derives the well known infogram.

The aim of the present research is to propose a novel frequency decomposition called "detectogram", based on the maximization of a Detectivity redefinition. In [9] Cocconcelli et al. derived the Detectivity by a proper combination of the Hjorth's parameters, namely activity, mobility and complexity. They are statistical time-domain parameters introduced by Hjorth and Elema-Schonander in 1970 [10], which are commonly used in different area of signal processing i.e., in the analysis of electroencephalography signals and in robotic area. Cocconcelli et al. derive the Detectivity with the aim of studying the evolution of a bearing fault during the full life-cycle of a mechanical system. In that context, Hjorth's parameter where mixed together and normalized by their mean values evaluated when the system was in sound conditions. The drawback of this definition concerns

the analysis of vibration signals in the absence of healthy historic data. In this work firstly, a redefinition of the Detectivity parameter is proposed, and then a frequency signal decomposition is derived. This signal decomposition is applied to bearing fault diagnostics. Both real and synthesized signals were taken into account.

The paper is structured as follows: Section 2 introduces the definition of the Detectivity parameter as well as its Spectral counterpart. Section 3 concerns the results on both synthesized and experimental data and Section 4 draws conclusions.

2 From Hjorth's parameters to Spectral Detectivity

In 1970, Hjorth and Elema-Schonander [10] introduced three parameters to characterize the shape of an ECG signal in medicine. Let $x(t)$ be a vibration signal, Hjorth's parameters are defined as:

- *Activity (Act)*: variance of $x(t)$;
- *Mobility (Mob)*: square root of the ratio between the Activity for the time-derivative of $x(t)$ and the Activity of $x(t)$ itself;
- *Complexity (Comp)*: ratio between the Mobility of the time-derivative of $x(t)$ and the Mobility of $x(t)$ itself.

Therefore:

$$Act(x(t)) = \sigma^2(x(t)); \quad Mob(x(t)) = \sqrt{\frac{Act(\dot{x}(t))}{Act(x(t))}}; \quad Comp(x(t)) = \frac{Mob(\dot{x}(t))}{Mob(x(t))} \quad (1)$$

The Detectivity parameter is defined by a proper combination of Hjorth's parameters with the idea of creating an indicator, which could span the entire life of a mechanical component highlighting the deterioration near the end of its life-cycle as:

$$Detectivity_{(dB)}(x(t)) = Act_{(dB)}(x(t)) - Mob_{(dB)}(x(t)) + Comp_{(dB)}(x(t)) \quad (2)$$

where the suffix (dB) means that the Horth's parameters were scaled to their corresponding dB values with respect to proper reference values. Such reference values correspond to the mean values of the parameters themselves when the mechanical component under test is in sound conditions. In this way, the obtained parameter can be extremely useful in monitoring life trend, whilst is useless for a single signal investigation due to the fact that the reference values are not defined. In order to overcome this limitation, a modified version of the Detectivity parameter is hereafter proposed.

In this work, the Hjorth's parameter are not scaled to their dB values, but a signal sampled from a Gaussian distribution with zero mean and unitary variance is chosen as a reference. This approach is designed toward the understanding of how information are carried inside signals [11]. Let $x(t)$ be a vibration signal and $s(t)$ a sampled version from a Gaussian distribution with zero mean and unitary variance, the proposed Detectivity parameter is defined as:

$$Detectivity(x(t)) = \frac{Act(x(t))}{Act(s(t))} \cdot \frac{Mob(s(t))}{Mob(x(t))} \cdot \frac{Comp(x(t))}{Comp(s(t))} \quad (3)$$

This approach allows the Detectivity parameter to be used in analysing single vibration signal as well as in extracting the fault trend for a complete life-cycle of the mechanical component under test. Moreover, by following the path of informative decomposition, a Spectral Detectivity can be obtained by computing the Detectivity at the output of a filter-bank at each frequency f [12]. As a matter of fact, a "detectogram" can be obtained by evaluating the Spectral Detectivity values for different combinations of the $(f, \Delta f)$ plane [12]. As stated by Antoni in [8], the complete exploration of the whole $(f, \Delta f)$ plane is a cumbersome task. Therefore, the detectogram could be evaluated over a dyadic grid in the $(f, \Delta f)$ plane, by using a Discrete Wavelet Transform (DWT) or a Discrete Wavelet Packet Transform (DWPT). For a dyadic decomposition, a short description of the algorithm is as follows:

1. Chose the decomposition level k , each levels will have 2^k bands;
2. Filter the signal at each decomposition level with a central frequency $f_i = (i + 2^{-1})2^{-k-1}$ with a bandwidth $\Delta f_k = 2^{-k-1}$, with $i = 0, \dots, 2^k - 1$;
3. The detectogram is computed by evaluating the Detectivity of all sequences with Equation (3).

In this work, Authors preferred the use of the 1/3-binary tree estimator based on the Short-time Fourier transform (STFT), more details can be found in [8].

3 Numerical examples and data analysis

In this section, the effectiveness and sensitivity of the proposed indicator is assessed on the basis of both numerical and experimental data.

3.1 Numerical examples

Firstly, the ability of the proposed parameter to describe a cyclostationary signal with respect to a Gaussian one is tested. In order to do that, a numerical example developed by Antoni and Borghesani in [3] is used. Two sets of numerically generated signals are used, the first one concerns identically distributed Gaussian signals (GS),

$$\mathbf{x}^{(r)} \sim \mathcal{N}(\mathbf{x}; 0, \sigma^2 \mathbf{I}), \quad r = 0, \dots, R-1 \quad (4)$$

whilst the second one regards a series of Gaussian Cyclostationary signals (GCS),

$$\mathbf{x}^{(r)} \sim \mathcal{N}(\mathbf{x}; 0, \text{Diag}\{\sigma^2(n)\}), \quad r = 0, \dots, R-1 \quad (5)$$

with an increasing modulation depth m_r :

$$\sigma_r^2(n) = \frac{\sigma^2(1 + m_r \sin(\frac{2\pi n}{N}))}{1 + m_r}, \quad r = 0, \dots, R-1 \quad (6)$$

The aforementioned numerical example is performed with $L = 10000$, $N = 100$ and $\sigma^2 = 3$, see [3] for more details. Figure 1 shows the evaluation of the Hjorth's parameters on the GS and GCS signals as well as the respectively Detectivity values computed via Equation (3). As expected the Activity parameter is strongly related to the variation of the signal variance (Figure 1(a)), on the contrary both Mobility and Complexity still remain constant regardless the signal cyclostationary content. However, due to the proposed combination of Hjorth's parameters, the Detectivity could tracks the cyclostationary signal content.

Lastly, the detectogram sensitivity to impulsive noise is investigated. For this purpose, a simulated signal composed by a cyclostationary train of impulse responses buried in a Gaussian background noise and a single impulsive event is generated. The synthesized signal is generated according to the model in [6, 13] via the convolution of the impulse response of single-degree-of-freedom system with normalized resonance frequency at 0.2 ($Fs = 1$) and damping ratio of 8.3% to a series of Dirac's with mean spacing $N = 120$ and 5% random jitter. The same model is applied for generating the impulsive noise, via the convolution of the impulse response of a single-degree-of-freedom system, with normalized resonance frequency 0.35 and damping ratio of 1.6% to a single Dirac located at time sample $n = 1000$. Such a signal is known to simulate rather realistically the vibration signature of a faulted rolling element bearing. The generated data are depicted in Figure 2(a) and (b) after the addition of white Gaussian noise with SNR = -12dB. Figure 2(c) plots the corresponding Spectral Coherence which highlights the resonance that carries the series of cyclostationary transients at $f = 0.2$ and the resonance of impulsive noise at $f = 0.35$.

Figure 3 shows the Spectral Hjorth's parameter estimated with STFT over 9 level 1/3-binary tree. It is possible to see that both Activity and Complexity can highlight the resonance frequency involving the cyclostationary content around $f = 0.2$ at level 9^{th} . Moreover, they are insensitive to the impulsive noise at $f = 0.35$.

The combination of the previous spectral indicator via the use of Equation (3) gives the detectogram depicted in Figure 4(a). In this case, both Activity and Complexity combines together highlighting the presence of the cyclostationary source, whilst the Mobility acts only as a scale parameter. Such result is pointed out

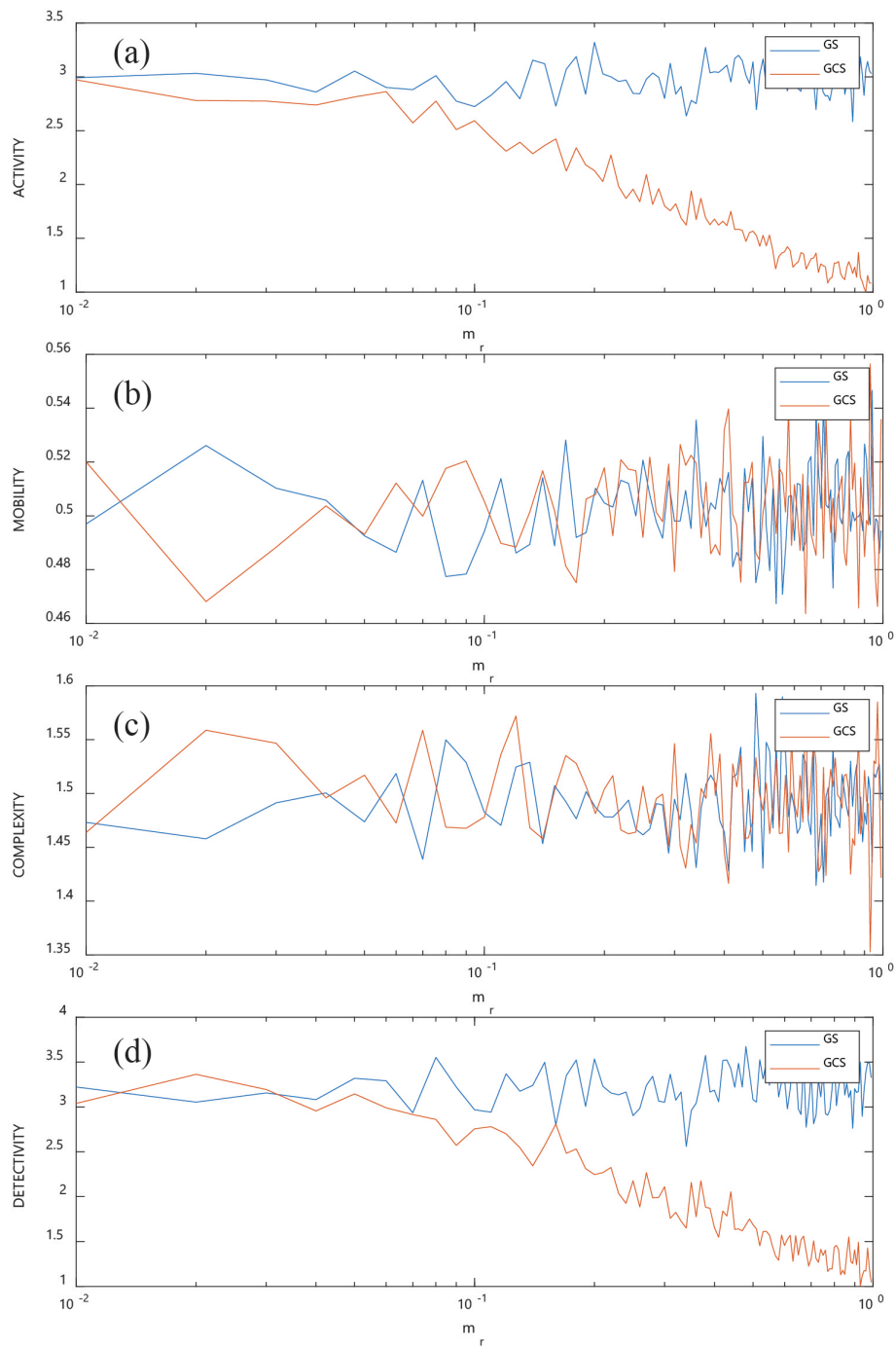


Figure 1: GS and GCS simulated signals: (a) Activity, (b) Mobility, (c) Complexity and (d) Detectivity

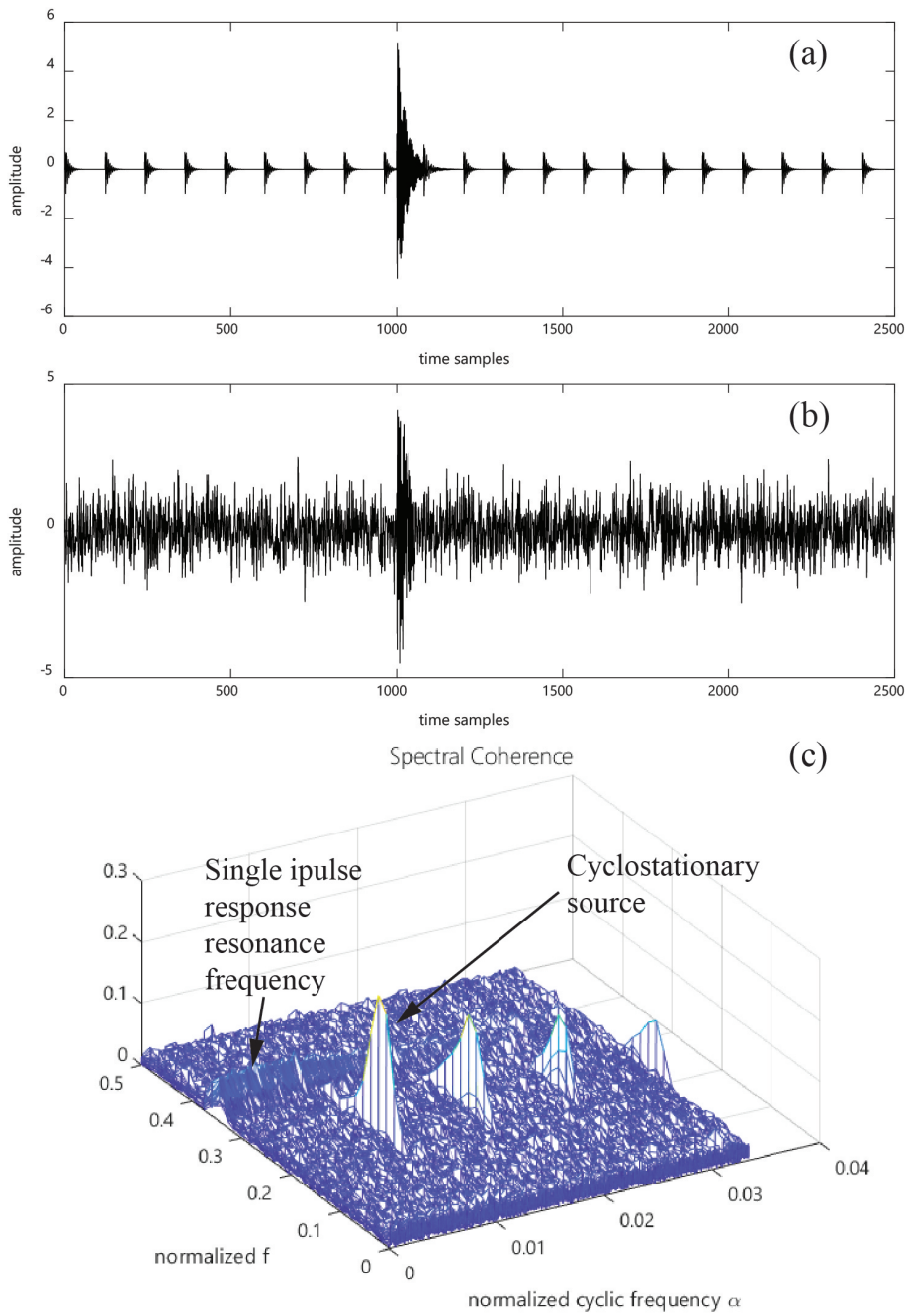


Figure 2: Numerically generated signal composed by a cyclostationary train of impulse responses and a single impulsive event: (a) without noise, (b) with white Gaussian noise, (c) Spectral Coherence of the noisy signal

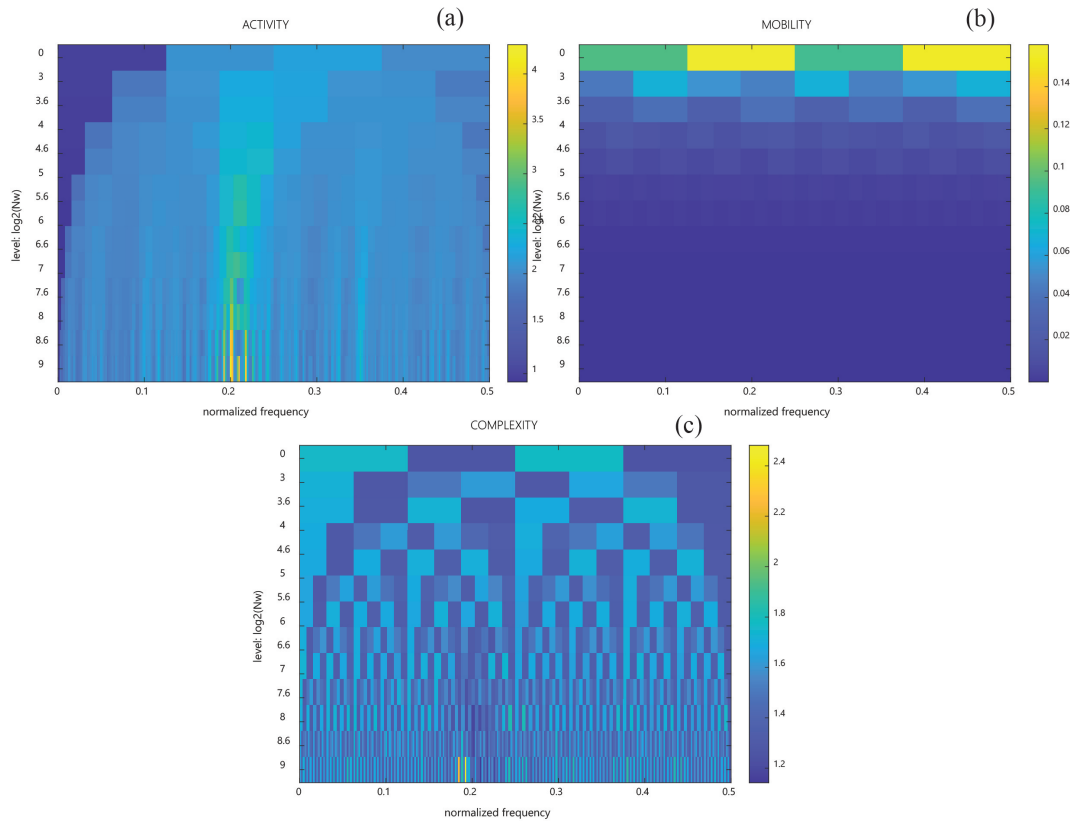


Figure 3: Numerically generated signal with impulsive noise: (a) Spectral Activity, (b) Spectral Mobility and (c) Spectral Complexity

in Figure 4(b), which shows the detectogram slice at level 9. As a matter of fact, the chosen combination of Hjorth's parameter are not indicative of the existence of the informative events with different structures i.e. cyclostationary and impulsive noise, but such result could be pivotal in some diagnostic applications on which the faulted signature is related to a cyclostationary content and the impulsive noise is not carrying any useful information about the health state of the machine.

3.2 Experimental data

This section concerns the diagnostics of real bearing fault with the proposed Detectivity parameter. In this work the bearing data of the Polytechnic of Turin (POLITO) test rig are taken into account [14]. In particular, the POLITO data-set consist of two main subset. The first one concerns different localized bearing damages running at different speed and loads, whilst the second one reports the behaviour of a single damaged bearing undergoing a long test at a constant speed and load. In this work the last subset, which covers the lifetime of a bearing (about 330h), is taken into account. A tested bearing is mounted on a high-speed spindle driven at a nominal constant speed of 18000 rpm. During test a spring mechanism applied a nominal radial load of 1800N to the bearing. The lubrication is carried out through an oil circuit system that regulated the temperature and flow ratio of the lubricant itself. Vibration signals were acquired every 30min for a time span of 8s via a piezoelectric accelerometer mounted on the bearing housing. More details on the test can be found in [14]. In the endurance test a pre-faulted bearing was used. In particular, the fault on the roller was made by a conical indentation with a maximum diameter of $450\mu\text{m}$ and mounted on the test rig after a break-in period. That means that there is no available healthy state data, but it possible to observe the evolution of fault severity in the bearing. The fault evolution was visually inspected three times during test at 70, 144 and 332h, corresponding to acquisition number 19, 34 and 66 respectively.

Figure 5 plots the Detectivity parameter evaluated with Equation (3). It is worth noting an abrupt change of the Detectivity parameter at the first inspection time, this behaviour is probably related to disassembly and reassembly operations of the bearing on the test rig. Notwithstanding, it is possible to see that Detectivity is

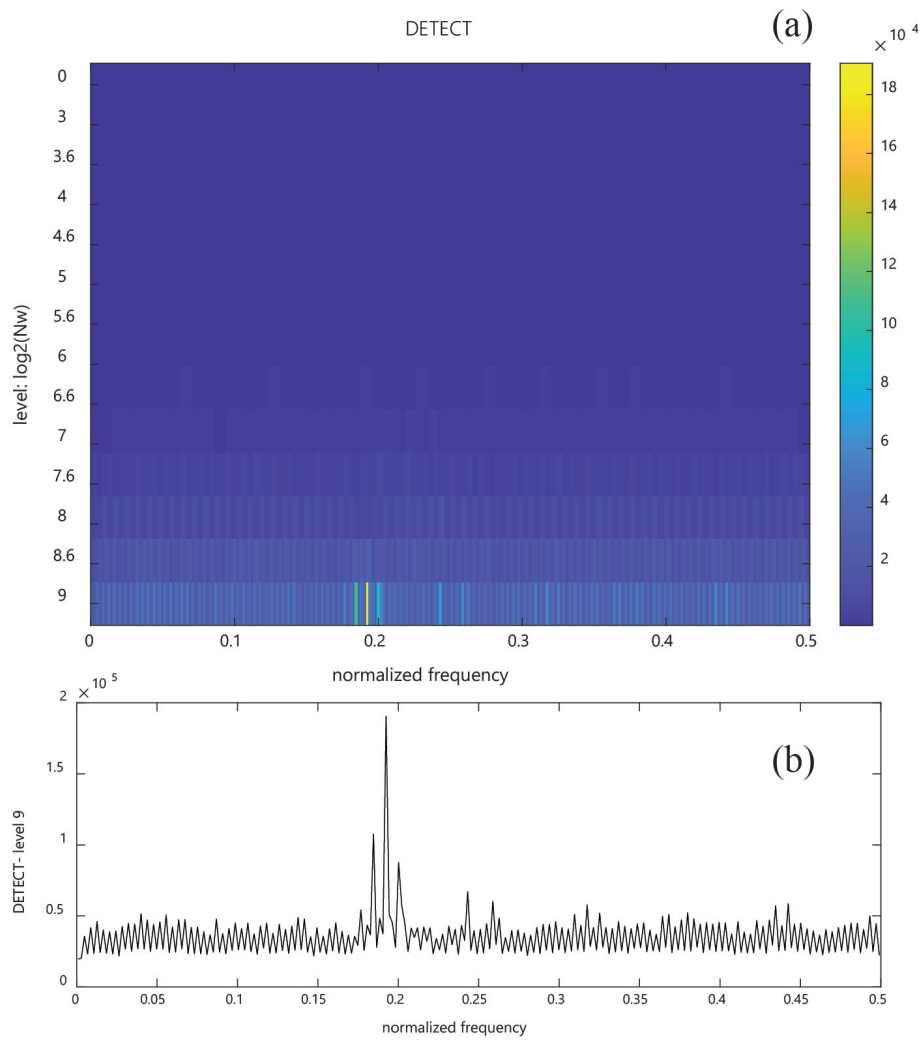


Figure 4: Numerically generated signal with impulsive noise: (a) detectogram, (b) frequency slice at level 9

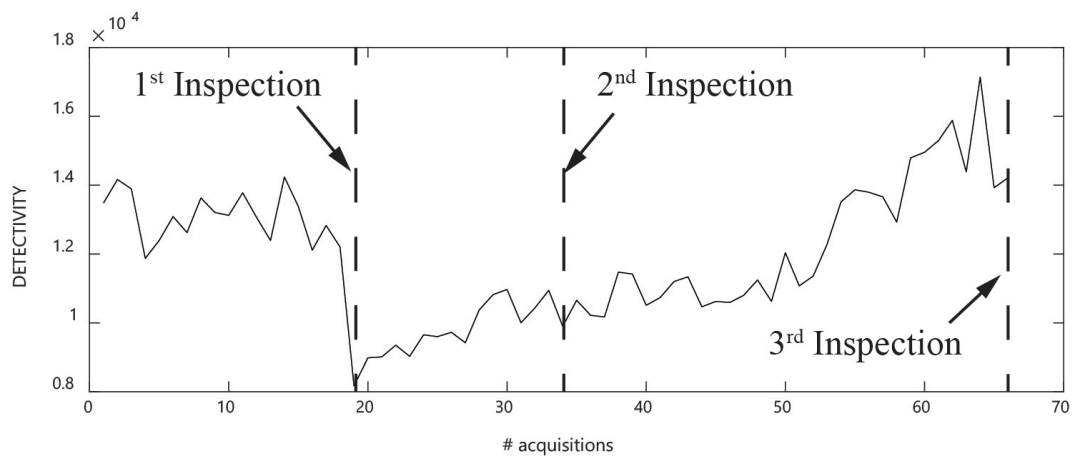


Figure 5: POLITO dataset: Detectivity trend

constantly increasing during test, highlighting therefore the capabilities of such parameter in tracking the fault evolution.

4 Concluding remarks

A redefinition of the Detectivity parameter proposed by Cocconcelli et al. [9] and a frequency signal decomposition named detectogram is presented in this paper. The effectiveness and sensitivity of such a signal decomposition to bearing fault diagnostics has been investigated based on both synthetic and real bearing vibration signals. In particular, the POLITO data-set is taken into account, which deal with the evolution of a bearing faults at constant speed and load. From the analysis of synthetic data it is possible to see that the proposed indicator is not sensitive to informative events with different structures i.e., cyclostationary and impulsive noise, but only to the cyclostationary ones. As a matter of fact, in bearing diagnostics this could be an interesting result due to the cyclostationary nature of the fault itself. Moreover, the ability of proposed Detectivity to track the fault evolution has been highlighted by the use of the POLITO data-set. The analysis of other type of faults will be the subject of future works.

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